Institute of Paper Science and Technology
Atlanta, Georgia

IPST Technical Paper Series Number 722

A Note on Helmholtz Attenuators with Air Cavity and Membrane

X. Wang and Z. Feng

April 1998

Submitted to
ASME Winter Meeting and Symposium
November 1, 1998

INSTITUTE OF PAPER SCIENCE AND TECHNOLOGY
PURPOSE AND MISSIONS

The Institute of Paper Science and Technology is a unique organization whose charitable, educational, and scientific purpose evolves from the singular relationship between the Institute and the pulp and paper industry which has existed since 1929. The purpose of the Institute is fulfilled through three missions, which are:

- to provide high quality students with a multidisciplinary graduate educational experience which is of the highest standard of excellence recognized by the national academic community and which enables them to perform to their maximum potential in a society with a technological base; and

- to sustain an international position of leadership in dynamic scientific research which is participated in by both students and faculty and which is focused on areas of significance to the pulp and paper industry; and

- to contribute to the economic and technical well-being of the nation through innovative educational, informational, and technical services.

ACCREDITATION

The Institute of Paper Science and Technology is accredited by the Commission on Colleges of the Southern Association of Colleges and Schools to award the Master of Science and Doctor of Philosophy degrees.

A Note on Helmholtz Attenuators with Air Cavity and Membrane

by

Xiaodong Wang † and Zhigang Feng ‡

Institute of Paper Science and Technology

500 10th St., N.W., Atlanta, GA 30318

Abstract

In this paper, we present an analytical expression for the resonance frequency of Helmholtz attenuators with air cavity and membrane. We derive the characteristic equations based on one-dimensional isentropic acoustic models and illustrate the effects on the resonance frequency of certain design variations.

† Assistant Professor, IPST

‡ Research Associate, IPST

Submitted to:

1998 ASME Winter Meeting Symposium Program Noise and Vibration Control

1 Introduction

The development of high speed paper machines requires a better control of pressure pulsation in approach flow systems. Current design of pressure pulsation attenuators can reduce significantly pressure variations, however, they have also been found ineffective in dealing with low frequency pulsations (< 1 Hz). The basic design concept of the attenuator used in the paper industry is that of the Helmholtz resonator. The characteristic property of such a resonator is its ability to absorb effectively pressure waves around a particular frequency, i.e., the so-called resonant frequency. As one of the fundamental acoustic devices, the Helmholtz resonators have been studied extensively [2] [3] [5]. The purpose of this paper is to present an analytical study of the resonance frequency for a particular Helmholtz resonator used in the paper industry.

2 Governing Equations

We model the Helmholtz resonator (as depicted in Fig. 1), with one-dimensional isentropic acoustic models, in which the air cavity is separated from the neck of water and the main pipe flow by a membrane. The membrane with the cross-sectional area A_2, thickness d, Young's modulus E_o, and density ρ_o is modeled as a simple spring-mass system with stiffness K_o and mass M_o. We also denote the cross-sectional area, length, displacement, pressure, density, bulk modulus, wave number, and wave speed as A_i, L_i, w_i, p_i, ρ_i, β_i, k_i, and c_i, where $i = 1$ and 2, representing the air and the water, respectively. In addition, we define the air cavity volume as $V_1 = A_1 L_1$ and the water neck volume as $V_2 = A_2 L_2$. From the constitutive relation $p_i = -\beta_i \dfrac{\partial w_i}{\partial x}$ and the linear momentum equation $\rho_i \dfrac{\partial^2 w_i}{\partial t^2} = -\dfrac{\partial p_i}{\partial x}$, we obtain the following governing equations

$$\frac{\partial^2 w_1}{\partial x^2} = \frac{1}{c_1^2}\frac{\partial^2 w_1}{\partial t^2}, \quad 0 \leq x \leq L_1 \tag{1}$$

$$\frac{\partial^2 w_2}{\partial x^2} = \frac{1}{c_2^2}\frac{\partial^2 w_2}{\partial t^2}, \quad L_1 \leq x \leq L_1 + L_2 \tag{2}$$

Furthermore, the characteristic solutions for displacements w_1 and w_2 can be written as:

$$w_1 = a_1 e^{-j(\omega t + k_1 x)} + b_1 e^{-j(\omega t - k_1 x)}, \quad 0 \leq x \leq L_1 \tag{3}$$

$$w_2 = a_2 e^{-j(\omega t + k_2 x)} + b_2 e^{-j(\omega t - k_2 x)}, \quad L_1 \leq x \leq L_1 + L_2 \tag{4}$$

where ω stands for the system natural frequency (angular frequency) and the four constants a_1, a_2, b_1 and b_2 can be determined from the fixed boundary condition at $x = 0$, the zero pressure boundary condition at $x = L_1 + L_2$, and the kinematic and dynamic matching conditions at $x = L_1$, i.e.,

$$\begin{aligned}
w_1 &= 0, & \text{at } x = \ & 0 \\
A_1 \frac{\partial w_1}{\partial t} &= A_2 \frac{\partial w_2}{\partial t}, & \text{at } x = \ & L_1 \\
A_2(-\beta_1 \frac{\partial w_1}{\partial x} + \beta_2 \frac{\partial w_2}{\partial x}) &= K_o w_2 + M_o \frac{\partial^2 w_2}{\partial t^2}, & \text{at } x = \ & L_1 \\
\frac{\partial w_2}{\partial x} &= 0, & \text{at } x = \ & L_1 + L_2
\end{aligned} \tag{5}$$

By introducing the dispersion relation $k_i = \frac{\omega}{c_i}$, with $c_i = \sqrt{\frac{\beta_i}{\rho_i}}$, we obtain the following characteristic equation

$$\frac{\beta_1 A_2}{A_1}\frac{\omega}{c_1} ctan(\frac{\omega}{c_1}L_1) = \beta_2 \frac{\omega}{c_2} tan(\frac{\omega}{c_2}L_2) - \frac{(K_o - \omega^2 M_o)}{A_2} \tag{6}$$

If we consider the same acoustic media (e.g. water) in both the cavity and the neck and ignore membrane effects, Eq. (6) becomes,

$$\frac{A_2}{A_1} ctan(\frac{\omega}{c_2}L_1) = tan(\frac{\omega}{c_2}L_2) \tag{7}$$

which is the same characteristic equation discussed in Ref. [6].

Since we are interested in the low resonance frequency, i.e., the coupled system natural frequency, we assume $k_i L_i = \frac{\omega}{c_i} L_i \ll 1$, which implies that the typical wave length is much larger than the cavity and orifice dimensions. Therefore, Eq. (6) is simplified as

$$\frac{\beta_1 A_2}{A_1}\frac{\omega}{c_1} = \frac{\omega}{c_1}L_1\beta_2\frac{\omega}{c_2}\frac{\omega}{c_2}L_2 - \frac{\omega}{c_1}L_1\frac{(K_o - \omega^2 M_o)}{A_2} \tag{8}$$

from which we derive the analytical expression for the natural frequency f,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\beta_1 A_2^2 + K_o V_1}{(\rho_2 V_2 + M_o)V_1}} \tag{9}$$

Interestingly, we find that Eq. (9) can also be derived from the force equilibrium at $x = L_1$ with a similar approach introduced in Ref. [4]. Assuming the membrane displacement is ξ or $w_2(L_1)$, the restoring force exerting on the membrane from the air cavity side is $-\frac{A_2\xi}{V_1}\beta_1 A_2$, and the inertia force contributed by the water neck is $-\rho_2 V_2 \ddot{\xi}$. Adding the spring-mass system of the membrane, we arrive at the following equation of motion,

$$(\rho_2 V_2 + M_o)\ddot{\xi} + (\frac{\beta_1 A_2^2}{V_1} + K_o)\xi = 0 \tag{10}$$

which gives us the natural frequency expression in Eq. (9).

Notice that the coupling between the air and the water does not introduce the complex wave number k_i or natural frequency ω, which indicates that no physical damping is introduced by the membrane. For the limiting case, if we neglect the existence of the membrane, i.e. $M_o = K_o = 0$, we obtain

$$f = \frac{c_1}{2\pi}\sqrt{\frac{\rho_1 A_2}{\rho_2 L_2 V_1}} \tag{11}$$

Furthermore, if the air cavity is filled with water, we recover the classical resonance frequency for the Helmholtz resonator [1]

$$f = \frac{c_2}{2\pi}\sqrt{\frac{A_2}{L_2 V_1}} \tag{12}$$

3 Numerical Results

To find the effects of various design variations on the resonance frequency, we test the example depicted in Fig. 1 with the following parameters: $\rho_1 = 1.21$ kg/m^3; $\rho_2 = 1000$ kg/m^3; $\beta_1 = 1.41 \times 10^5$ Pa; $\beta_2 = 2.1 \times 10^9$ Pa; $L_1 = 2$ m; $L_2 = 0.8$ m; $A_1 = 1$ m^2; $A_2 = 0.14$ m^2; $\rho_o = 1260$ kg/m^3; $E_o = 1.41 \times 10^9$ Pa; and $d = 0.005$ m. For the particular configuration described above, we obtain the spring-mass system constants $K_o = 7.05 \times 10^4$ N/m $= 1\%\, dE_o$, $M_o = d\rho_o A_2 = 0.882$ kg and $\omega_o = \sqrt{\frac{K_o}{M_o}} = 282.72$ rad/sec. From Eq. (9), we can calculate the following resonance frequencies:

$$\begin{aligned} f &= 4.02 \text{ Hz}, && \text{With the membrane effects} \\ f &= 0.56 \text{ Hz}, && \text{Neglect the membrane effects} \\ f &= 68.2 \text{ Hz}, && \text{Cavity filled with water.} \end{aligned}$$

Notice that the resonance frequency of the traditional Helmholtz resonator filled with water is significantly higher than that of the modified version with air cavity and membrane. In addition, we find that the net effect of the membrane leads to a resonance frequency augmentation, which is further confirmed in Fig. 3. Of course, in the actual design, we will never achieve the stable separation between the air and the water right at the loation of $x = L_1$, and the case in which we neglect the membrane effects gives us a lower bound for the resonance frequency.

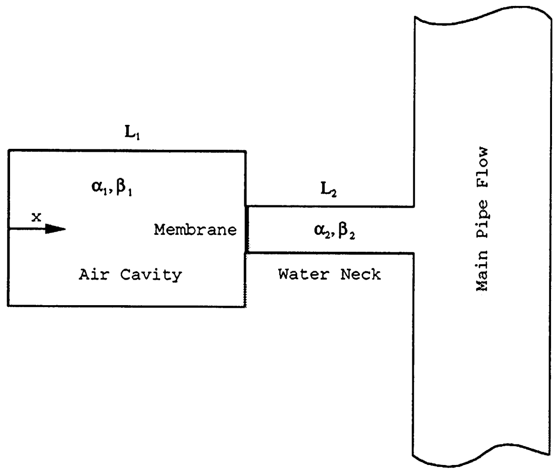

Figure 1: The Helmholtz resonator with air cavity and membrane.

As illustrated in Figs. 2 to 4, the resonance frequency can be significantly reduced by minimizing the membrane stiffness or increasing the neck length or the air cavity volume. Figure 4 also shows that if the membrane stiffness is dominant, the increase of the cross-sectional area ratio $\frac{A_2}{A_1}$ will reduce the resonance frequency, which is in contradiction with the design intuitions of the Helmholtz attenuators without membrane effects (refer to Eqs. (11) and (12)).

4 Conclusion

The main contribution of this paper is the derivation of an analytical expression for the resonance frequency of Helmholtz resonators with air cavity and membrane. We find that the one-dimensional isentropic acoustic model for both the air and the water and the lumped stiffness and mass model result in the same approximation of the resonance frequency. In addition, by varying different design parameters, we derive the following design guidelines:

(1) By increasing the neck length, we can decrease the natural frequency;

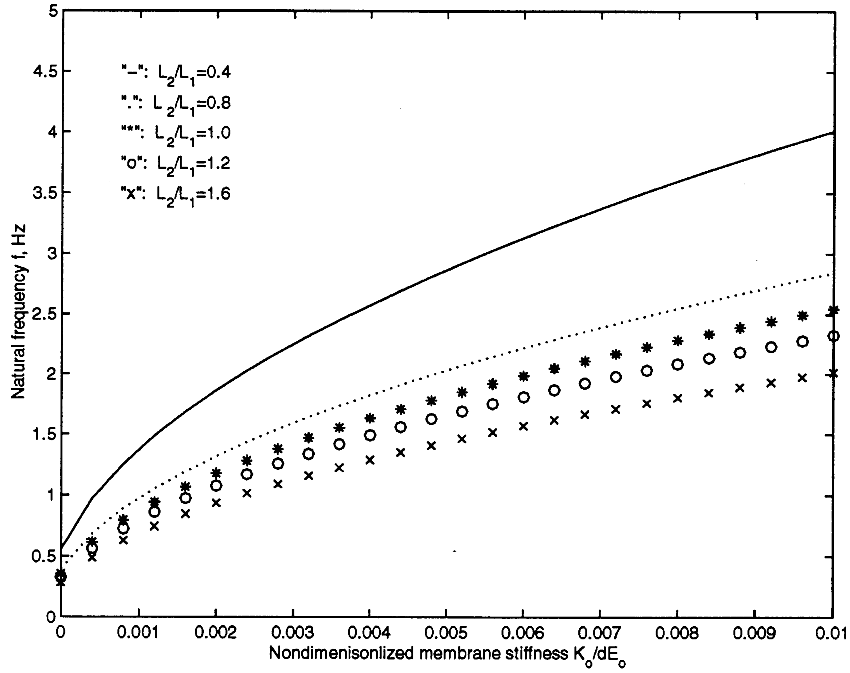

Figure 2: Resonance frequency *vs.* membrane stiffness.

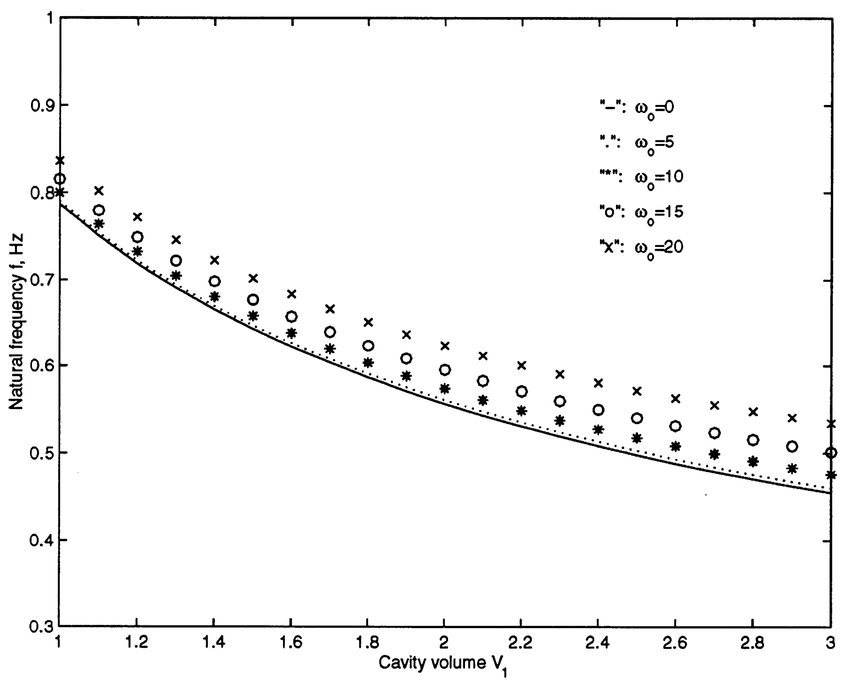

Figure 3: Resonance frequency *vs.* air cavity volume.

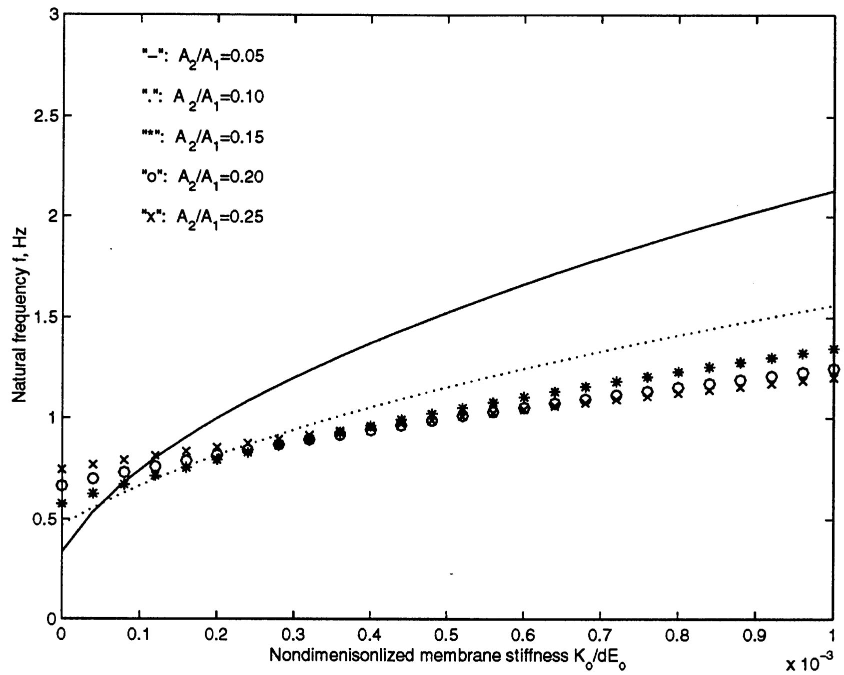

Figure 4: Resonance frequency *vs.* cross-sectional area ratio.

(2) To minimize the resonance frequency, the membrane natural frequency should be as low as possible without the detriment of the main function of separating the water flow from the air cavity;

(3) Increase the neck cross-sectional area does not necessarily reduce the resonance frequency. The effect depends on the membrane stiffness.

Future work is needed to consider the multidimensional effects with computational simulations.

5 Acknowledgment

The author would like to thank the Institute of Paper Science and Technology and its Member Companies for their support.

References

[1] R.D. Blevins. *Formulas for Natural Frequency and Mode Shape.* Van Nostrand Reinhold Company, 1979.

[2] Uno Ingard. On the theory and design of acoustic resonators. *Journal of Acoustic Society of America*, 25(6):1037–1061, 1953.

[3] L.E. Kinsler, A.R. Frey, A.B. Coppens, and J.V. Sanders. *Fundamentals of Acoustics.* McGraw-Hill Publishing Company, third edition, 1982.

[4] K.L. Koai, T. Yang, and J. Chen. The application of finite element method in analyzing the noise reduction effect of Helmholtz resonator attachments. *Proceedings of the ASME Noise Control and Acoustics Division*, 22:199–205, 1996.

[5] P.M. Morse. *Vibration and Sound.* McGraw-Hill Publishing Company, second edition, 1948.

[6] A. Selamet, N.S. Dickey, and J.M. Novak. Theoretical computational and experimental investigation of Helmholtz resonators with fixed volume: Lumped versus distributed analysis. *Journal of Sound and Vibration*, 187(2):358–367, 1995.